Indian Statistical Institute, Bangalore
B. Math (II), Second Semester 2017-18
MidSemester Examination : Statistics (II)

Date: 01-03-2018 **Maximum Score 40** **Duration: 3 Hours**

1. Let $X_1,\ X_2, \cdots, X_n$ be a random sample from the distribution specified by

$$f(x,\theta) = e^{-(x-\theta)} I_{(\theta,\ \infty)}(x);\ x, \theta \in I\!R.$$

Let $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, $X_{(1)} = \min\limits_{1\le i\le n} X_i$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$.

Obtain $\Pr_\theta\left[X_{(1)} \le y | S^2 \ge s\right]$; $y, s \in I\!R$.

[12]

2. Number of particles emitted by a radioactive substance may be modelled by a *Poisson Process* with *parameter* λ, $\lambda > 0$. Here λ is the rate of emission per unit of time. A physicist conducting an experiment is interested in estimating $\psi(\lambda)$, the probability of at most k particles being emitted per unit of time. For small k large $\psi(\lambda)$ may be considered as a good measure of *nuclear safety*. Let $X_1,\ X_2, \cdots, X_n$ denote the number of particles emitted in one unit of time recorded on n different occasions during the experiment. Can you assist the physicist to estimate the required probability $\psi(\lambda)$? Give an unbiased estimator for $\psi(\lambda)$. Is your estimator *uniformly minimum variance unbiased estimator (UMVUE)* for estimating $\psi(\lambda)$? If yes, substantiate. If no, obtain *UMVUE* for $\psi(\lambda)$.

[2 + 2 + 8 = 12]

3. In the above question (2) suppose there are reasons to believe that the parameter λ of the *Poisson Process* may be modelled using a *prior distribution* $\pi(\lambda)$ given by *Gamma*(a, b), $a > 0$, $b > 0$ known. Obtain *posterior distribution* of λ given the observations $X_1, X_2, \cdots, X_n$. Obtain *posterior* mean. Explain *Bayes Risk and* obtain *Bayes Estimator* for λ. Does your estimator have any interesting interpretation?

[4 + 2 + 4 + 2 = 12]

4. The life span of a washing machine is *exp*(λ), $\lambda > 0$. Let $X_1,\ X_2, \cdots, X_n$ denote the life spans of n randomly chosen similar machines. Based on $X_1,\ X_2, \cdots, X_n$ derive *likelihood ratio test* (*LRT*) for the testing problem

$$H_0 : \lambda \ge \lambda_0 \ versus\ H_1 : \lambda < \lambda_0$$

such that it satisfies the probability of *type I error* at the threshold value λ_0 is exactly 0.05.

[12]